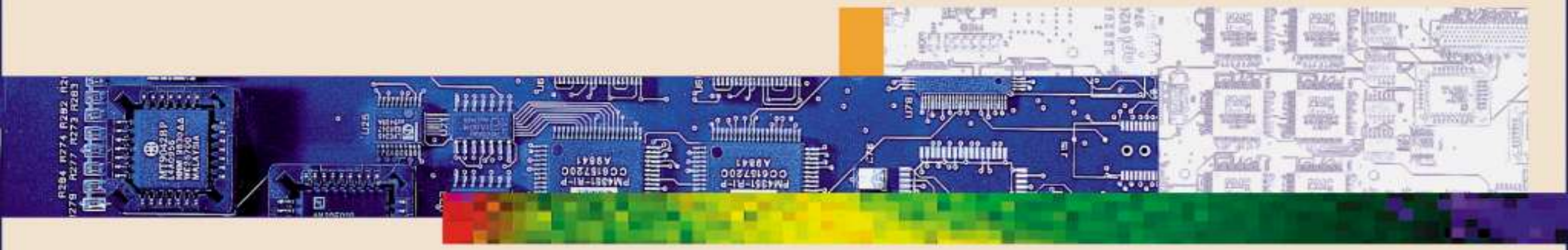

S-parameters
An introduction

Author: Jean Burnikell
Presented: Martyn Gaudion

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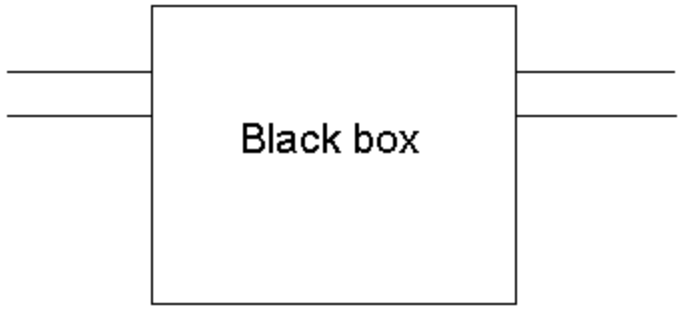
Polar

S-parameters are a useful method for representing a circuit as a “black box”



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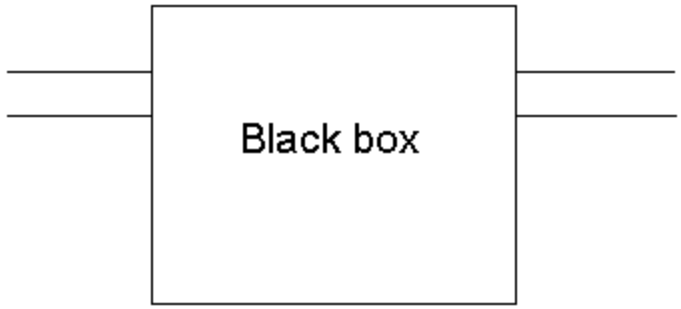
The external behaviour of this black box can be predicted without any regard for the contents of the black box.



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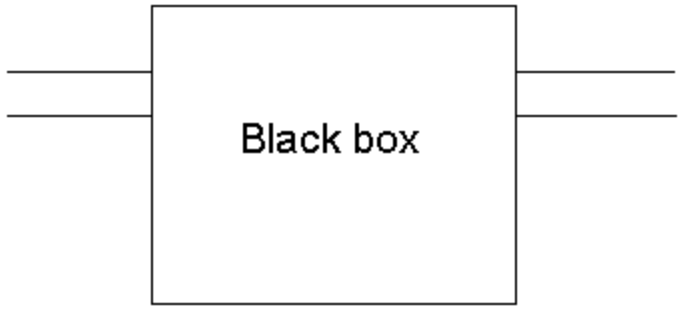
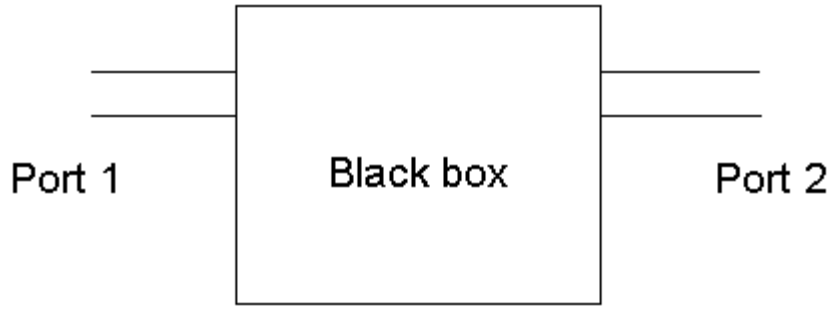


This black box could contain anything:

a resistor,
a transmission line
or an integrated circuit.

A “black box” or network may have any number of ports.

This diagram shows a simple network with just 2 ports.



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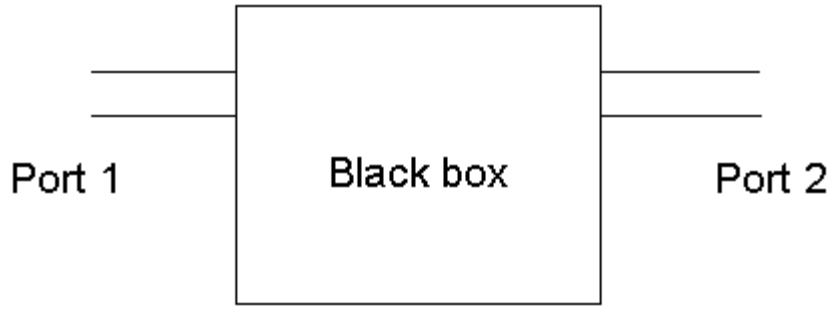
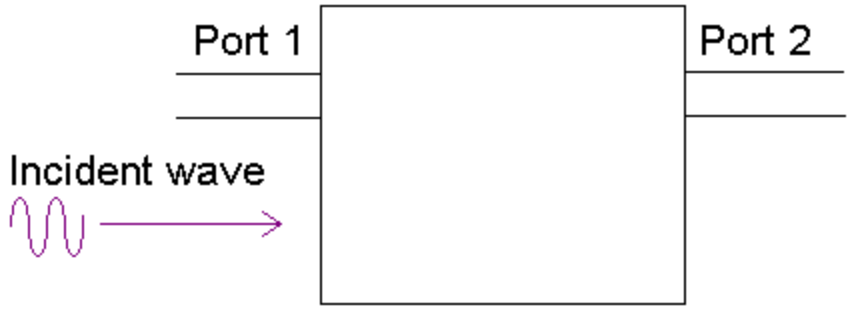


Note :

A *port* is a terminal pair of lines.

S-parameters are measured by sending a *single* frequency signal into the network or “black box” and detecting what waves exit from each port.

Power, voltage and current can be considered to be in the form of waves travelling in both directions.



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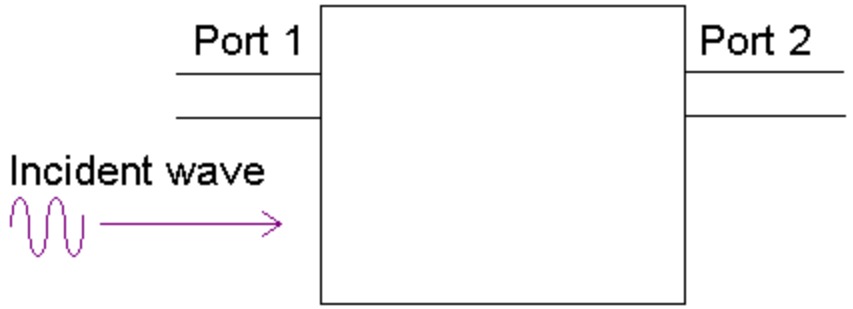
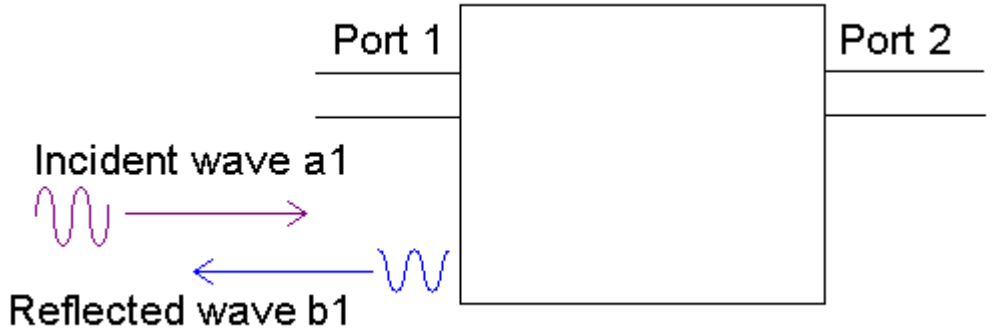


For a wave incident on Port 1, some part of this signal reflects back out of that port and some portion of the signal exits other ports.

I have seen S-parameters described as S_{11}, S_{21}, etc. Can you explain?

First lets look at S_{11}.

S_{11} refers to the signal reflected at Port 1 for the signal incident at Port 1.



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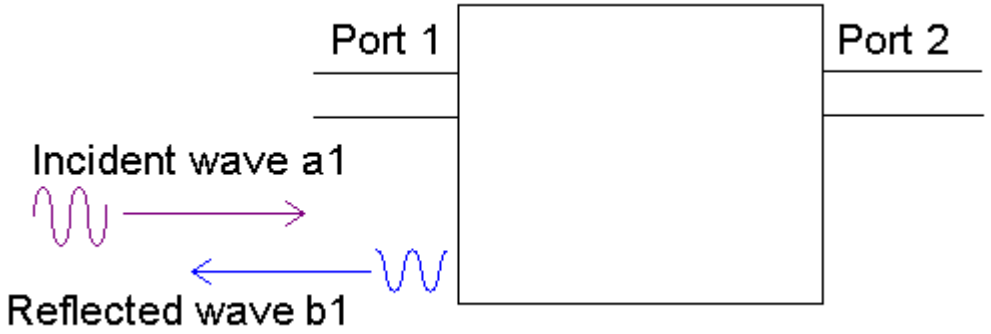

I have seen S-parameters described as S_{11}, S_{21}, etc. Can you explain?

Now lets look at S_{21}.

S_{21} refers to the signal exiting at Port 2 for the signal incident at Port 1.

Scattering parameter S_{21} is the ratio of the two waves b2/a1.

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S_{21} is correct! S-parameter convention always refers to the responding port first!

S22 refers to a signal exiting at Port 2 for an incident signal at Port 2

S22 is the ratio of b2/a2

S12 refers to a signal exiting at Port 1 for a incident signal at Port 2

S12 is the ratio of b1/a2

I have seen S-parameters described as S_{11}, S_{21}, etc. Can you explain?

A linear network can be characterised by a set of simultaneous equations describing the exiting waves from each port in terms of incident waves.

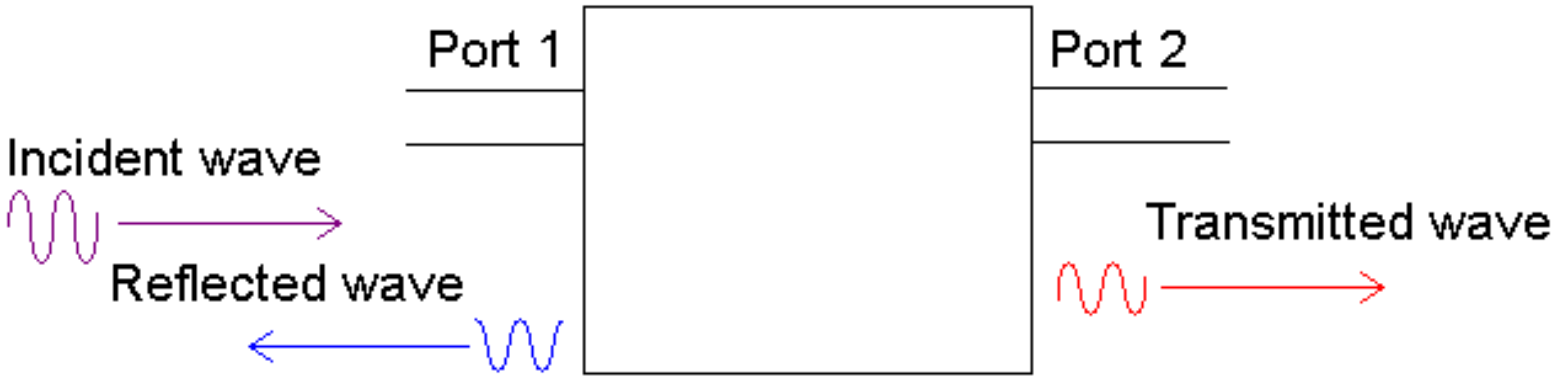
$S_{11} = b1 / a1$

$S_{12} = b1 / a2$

$S_{21} = b2 / a1$

$S_{22} = b2 / a2$

Note again how the subscript follows the parameters in the ratio (S_{11}=b1/a1, etc...)



The transmitted and the reflected wave will have changed in amplitude and phase from the incident wave.

Generally the transmitted and the reflected wave will be at the same frequency as the incident wave.

S-parameters are complex (i.e. they have magnitude and angle) because **both** the magnitude and phase of the input signal are changed by the network.

(This is why they are sometimes referred to as *complex scattering parameters).*

These four S-parameters actually contain eight separate numbers:

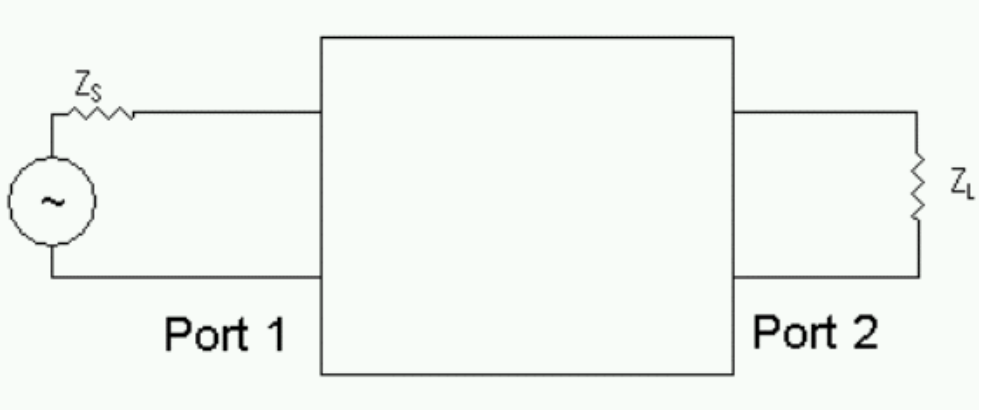
the real and imaginary parts (or the modulus and the phase angle)
of each of the four complex scattering parameters.

Quite often we refer to the magnitude only as it is of the most interest.

How much gain (or loss) you get is usually more important than how much the signal has been phase shifted.

What do S-parameters depend on?

S-parameters depend upon the network and the characteristic impedances of the source and load used to measure it, and the frequency measured at.



i.e.

if the network is changed, the S-parameters change.

if the frequency is changed, the S-parameters change.

if the load impedance is changed, the S-parameters change.

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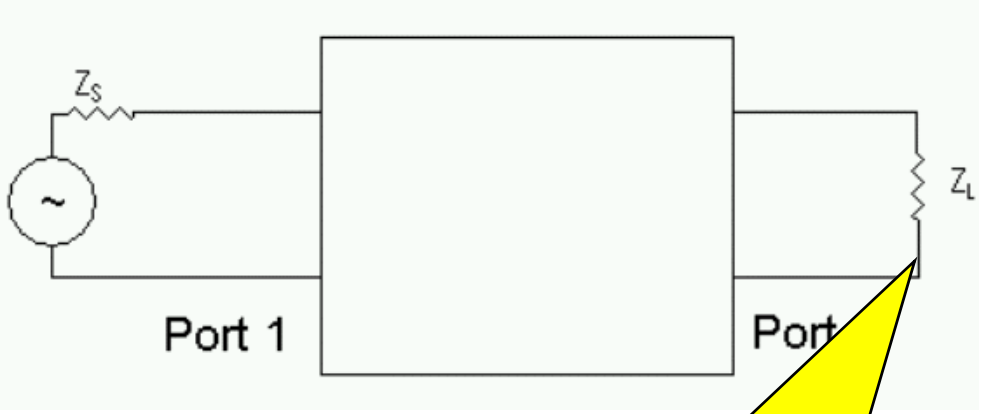

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In the Si9000e S-parameters are quoted with source and load impedances of 50 Ohms

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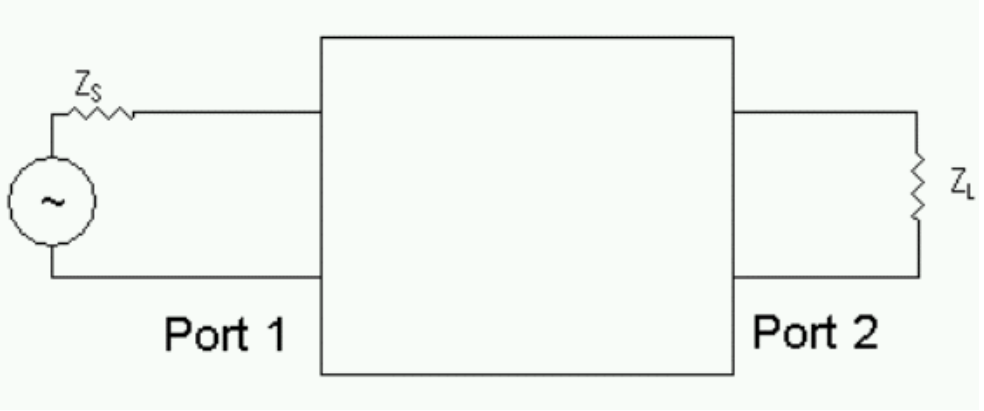

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A little math…

This is the matrix algebraic representation of 2 port S-parameters:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \times \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Some matrices are symmetrical. A symmetrical matrix has symmetry about the leading diagonal.

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In the case of a symmetrical 2-port network, that means that $S_{21} = S_{12}$ and interchanging the input and output ports does not change the transmission properties.

A transmission line is an example of a symmetrical 2-port network.

A little math…

Parameters along the leading diagonal, S_{11} & S_{22}, of the S-matrix are referred to as *reflection coefficients* because they refer to the reflection occurring at one port only.

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Off-diagonal S-parameters, S_{12}, S_{21}, are referred to as *transmission coefficients* because they refer to what happens from one port to another.

Larger networks:

A Network may have any number of ports.

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The S-matrix for an n-port network contains n^2 coefficients (S-parameters), each one representing a possible input-output path.

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For the S-parameter subscripts “ij”, “j” is the port that is excited (the input port) and “i” is the output port.

Larger networks:

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Yes i for **o**utput j for input
— logical ;-)

The S-matrix for an n-port network contains n² elements (S-parameters), each one representing a possible input-output path.

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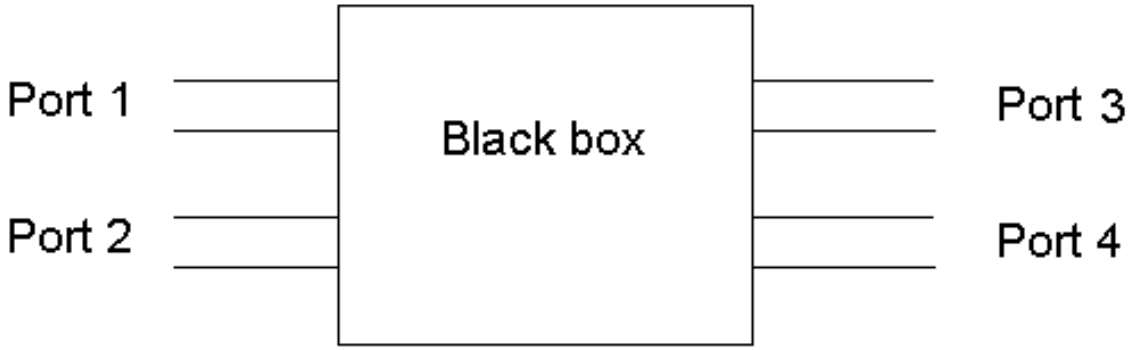
This is a 4-port network



$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \times \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Sum up…

- S-parameters are a powerful way to describe an electrical network
- S-parameters change with frequency / load impedance / source impedance / network
- S_{11} is the *reflection coefficient*
- S_{21} describes the *forward transmission coefficient* (responding port 1st!)
- S-parameters have both magnitude and phase information
- Sometimes the gain (or loss) is more important than the phase shift and the phase information may be ignored
- S-parameters may describe large and complex networks

- If you would like to learn more please see next slide:

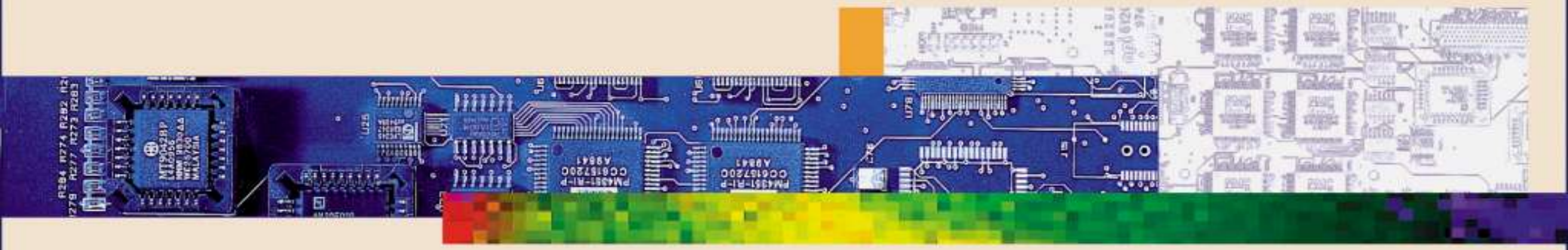
Further reading:

Agilent papers
http://www.sss-mag.com/pdf/an-95-1.pdf
http://www.sss-mag.com/pdf/AN154.pdf

National Instruments paper
http://zone.ni.com/devzone/nidzgloss.nsf/webmain/D2C4FA88321195FE8625686B00542EDB?OpenDocument

Other links:
http://www.sss-mag.com
http://www.microwaves101.com/index.cfm
http://www.reed-electronics.com/tmworld/article/CA187307.html
http://en.wikipedia.org/wiki/S-parameters

Online lecture OLL-140 Intro to S-parameters - Eric Bogatin
Online lecture OLL-141 S_{11} & Smith charts - Eric Bogatin
www.bethesignal.com



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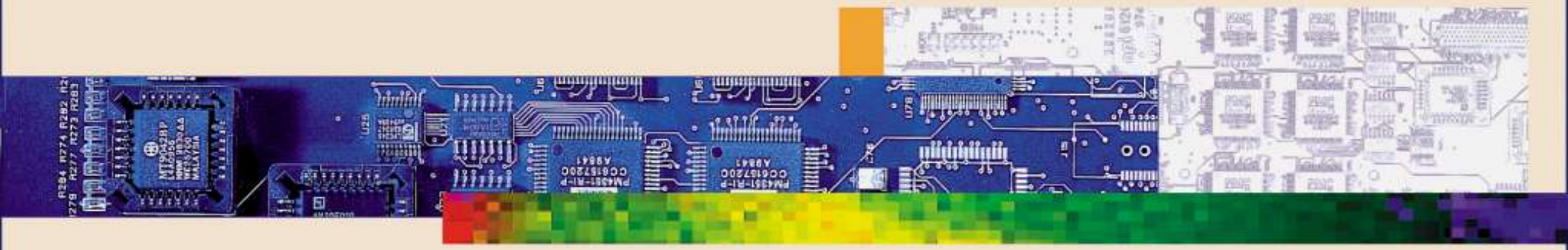

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